

Exercise 8

Evaluate the limit and justify each step by indicating the appropriate Limit Law(s).

$$\lim_{t \rightarrow 2} \left(\frac{t^2 - 2}{t^3 - 3t + 5} \right)^2$$

Solution

$$\lim_{t \rightarrow 2} \left(\frac{t^2 - 2}{t^3 - 3t + 5} \right) \left(\frac{t^2 - 2}{t^3 - 3t + 5} \right)$$

The limit of a product is the product of the limits.

$$\left[\lim_{t \rightarrow 2} \left(\frac{t^2 - 2}{t^3 - 3t + 5} \right) \right] \left[\lim_{t \rightarrow 2} \left(\frac{t^2 - 2}{t^3 - 3t + 5} \right) \right]$$

The limit of a quotient is the quotient of the limits, provided the denominator is not zero.

$$\left[\frac{\lim_{t \rightarrow 2} (t^2 - 2)}{\lim_{t \rightarrow 2} (t^3 - 3t + 5)} \right] \left[\frac{\lim_{t \rightarrow 2} (t^2 - 2)}{\lim_{t \rightarrow 2} (t^3 - 3t + 5)} \right]$$

The limit of a sum (difference) is the sum (difference) of the limits.

$$\left(\frac{\lim_{t \rightarrow 2} t^2 - \lim_{t \rightarrow 2} 2}{\lim_{t \rightarrow 2} t^3 - \lim_{t \rightarrow 2} 3t + \lim_{t \rightarrow 2} 5} \right) \left(\frac{\lim_{t \rightarrow 2} t^2 - \lim_{t \rightarrow 2} 2}{\lim_{t \rightarrow 2} t^3 - \lim_{t \rightarrow 2} 3t + \lim_{t \rightarrow 2} 5} \right)$$

The limit of a constant times a function is the constant times the limit of the function.

$$\left(\frac{\lim_{t \rightarrow 2} t^2 - \lim_{t \rightarrow 2} 2}{\lim_{t \rightarrow 2} t^3 - 3 \lim_{t \rightarrow 2} t + \lim_{t \rightarrow 2} 5} \right) \left(\frac{\lim_{t \rightarrow 2} t^2 - \lim_{t \rightarrow 2} 2}{\lim_{t \rightarrow 2} t^3 - 3 \lim_{t \rightarrow 2} t + \lim_{t \rightarrow 2} 5} \right)$$

Evaluate the limits.

$$\left[\frac{(2)^2 - 2}{(2)^3 - 3(2) + 5} \right] \left[\frac{(2)^2 - 2}{(2)^3 - 3(2) + 5} \right]$$

$$\left(\frac{2}{7} \right) \left(\frac{2}{7} \right)$$

$$\frac{4}{49}$$